FS2 Multiplication and Division

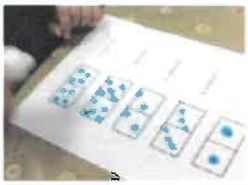
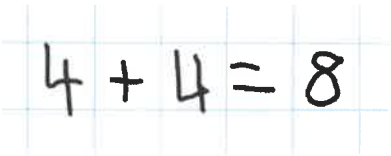
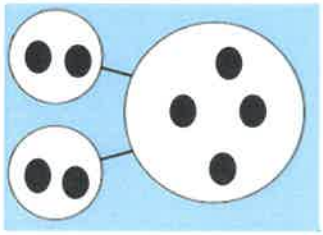
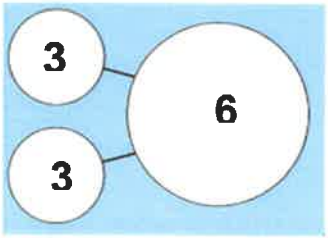
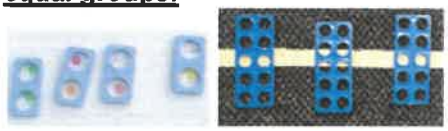
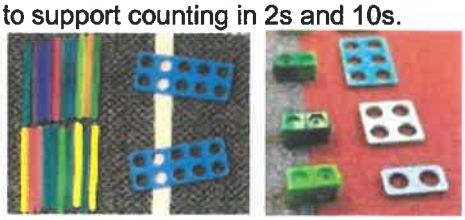
ELG:

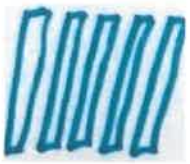
• Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.

Mental calculation:

Number facts:

Begin to count in 2s and 10s (where appropriate)

Concrete	Pictorial	Abstract
Real objects / manipulatives representing amounts for doubling and halving:    Use **real items** to double and halve amounts/objects physical. Use **real items** in a mirror to double.	   Art and craft activities involving doubling and halving, with basic recording of digits. 'Double..' and 'Half of…' used orally.	 Some children would be able to record doubling as addition by the end of the year.
Concrete	**Pictorial**	**Abstract**
Part Part Whole:   Begin to use 'Part part whole' model with **real items** and or Numicon to support halving. Emphasis on **equal** parts for halving.	 Begin to use 'Part part whole' model with drawing to support halving. Emphasis on **equal** parts for halving.	 Using the terms "**double**" and "**halve / half of**" orally, but children will not record this.
Real objects / manipulatives representing amounts for equal groups:   Use **real items** organised in 2s and 10s, to support counting in 2s and 10s.	  Drawings of **real items** given to children, to support counting in 2s or 10s.	$2 + 2 + 2 + 2 = 8$ Some children would be able to record multiple equal groups as repeated addition by the end of the year.
Concrete	**Pictorial**	**Abstract**
Numicon representing amounts for equal groups:  Use Numicon to represent 2s and 10s, to support counting in 2s and 10s.  Use **real items** alongside Numicon to support counting in 2s and 10s.	 Drawings of Numicon 2s or 10s given to children, to support counting in 2s or 10s.	As above

Concrete	Pictorial	Abstract
Diennes and money representing amounts for equal groups:   Use Diennes and 10p coins to represent 10s, to support counting in 10s.	  Drawings of Diennes 10s and 10p coins given to children, to support counting in 10s.  Children can draw diennes tens or 10p coins, to count in 10s.	As above

Y1 Multiplication and Division

(NC) Pupils should be taught to:

- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Mental calculation:

Number facts:

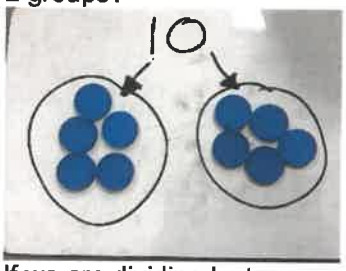
Counting in 2s, Counting in 10s, Counting in 5s

Doubles to 10 (and corresponding halving facts)

Concrete

Real objects / manipulatives representing amounts for sharing into equal groups:

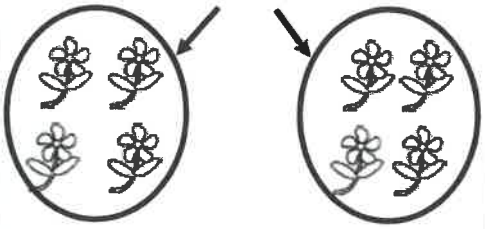
Sharing objects into groups- I have 10 cubes; can you share them equally into 2 groups?



If we are dividing by two, we are finding one half. It is important to make links with fractions.

Pictorial

Children use pictures or shapes to share quantities.



8 ÷ 2 = 4

Abstract

One half of 14 is 7

½ of 14 = 7

14÷2 = 7

Concrete

Real objects / manipulatives representing amounts for doubling:

Use practical activities to show how to double a number.

Double 5 is 10

Pictorial

Draw pictures to show how to double a number.

Double 4 = 8

Abstract

Double 4 = 8

Some children may be able to translate this to abstract recording with 'x' symbol:

4 x 2 = 8

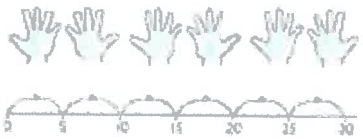
Concrete

Real objects / manipulatives representing amounts for equal groups:

Counting in multiples supported by concrete objects in equal groups.

Repeated addition - use different objects to add equal objects

Pictorial



Use a number line or pictures to continue support in counting in multiples.

5 lots of 2 is 10

Abstract

Count in multiples of a number aloud. Write sequences with multiples of numbers:

2, 4, 6, 8, 10…

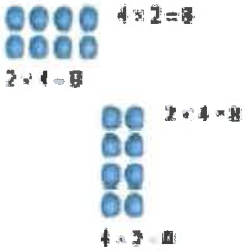
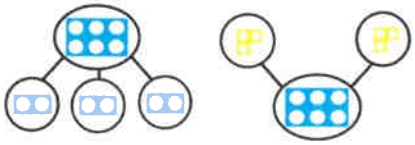
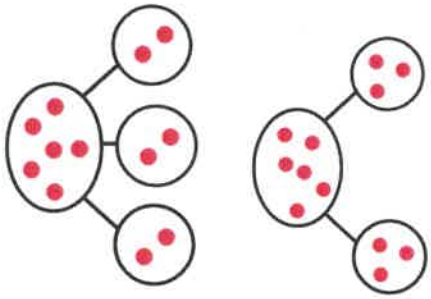
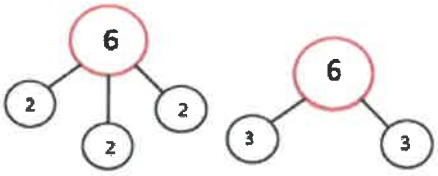
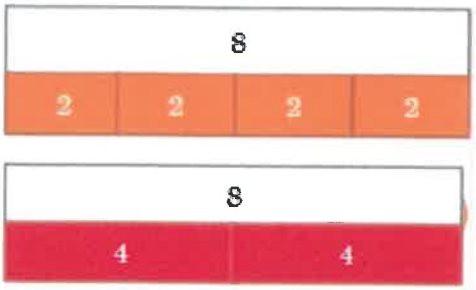
5, 10, 15, 20, 25, 30…

Write addition sentences to describe objects and pictures.

2+2+2+2+2=10

Some children may be able to translate this to abstract recording with 'x' symbol:

5 x 2 = 10

Concrete	Pictorial	Abstract
Real objects / manipulatives representing arrays:  Create arrays using counters/cubes to represent multiplication sentences. 5 lots of 3 = 15 3 lots of 5 = 15	 Draw arrays in different rotations to find commutative multiplication sentences.	Use an array to write multiplication sentences and reinforce repeated addition. 5 + 5 + 5 = 15 5 x 3 = 15 3 x 5 = 15
Concrete	**Pictorial**	**Abstract**
Part part whole:  Emphasis on parts being equal, use Numicon, counters or Cuisenaire with the model.	 Draw counters or around Numicon / Cuisenaire.	 2 + 2 + 2 = 6 3 lots of 2 = 6 3 x 2 = 6 3 + 3 = 6 2 lots of 3 = 6 2 x 3 = 6
Concrete	**Pictorial**	**Abstract**
Bar models: Emphasis on parts being equal  Use Cuisenaire to represent the multiple parts and whole of the bar model.	 2 + 2 + 2 + 2 = 8 4 lots of 2 = 8 Children draw using squared paper with one square = 1.	 Children record repeated addition using the bar model:

Y2 Multiplication and Division

(NC) Pupils should be taught to:

- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

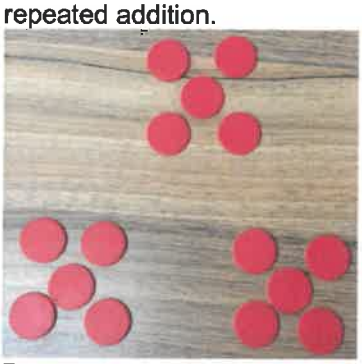
Mental calculation:

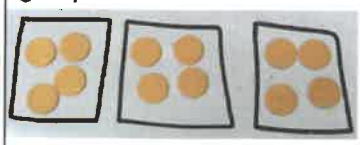
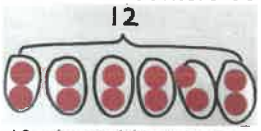
(NC) Pupils should be taught to:

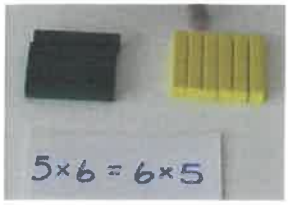
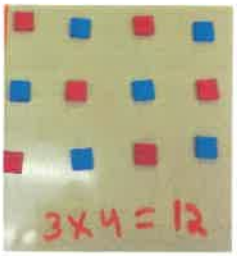
- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers

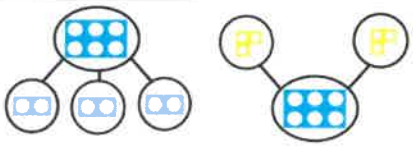
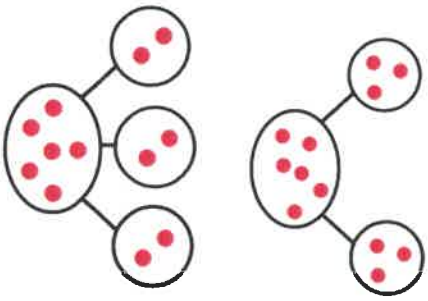
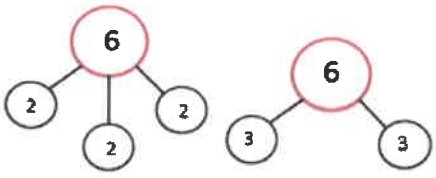
Number facts:

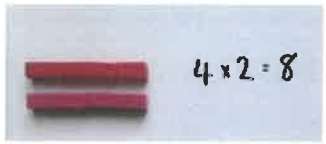
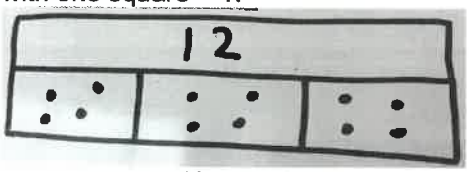
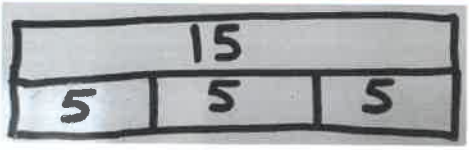
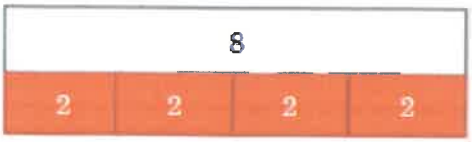
Know x and associated ÷ facts for 2 times table
Know x and associated ÷ facts for 10 times table
Know x and associated ÷ facts for 5 times table
Counting in 3s, Counting in 4s

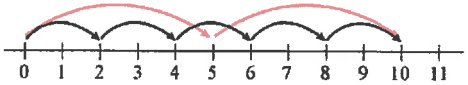
Concrete	Pictorial	Abstract
Real objects / manipulatives representing amounts for equal groups (x): Children can make equal groups using a variety of concrete resources to support conceptual understanding of 'times tables'.  They will add equal groups using repeated addition.  Represent multiplication problems using concrete resources. *There are 3 baskets.* *There are 5 apples in each basket.* *How many apples altogether?*	 Children draw their own objects to represent the numbers. '*5 x 4 = 20*'  Draw their own pictures to solve multiplication problems to reinforce language. *There are 3 baskets.* *There are 5 apples in each basket.* *How many apples altogether?*	Use repeated addition to represent the calculation. *5 + 5 + 5 + 5 = 20* Link the stem sentence, concrete resources and repeated addition to introduce the multiplication symbol. *5 X 4 = 20* *__ X __ = __* *__ lots of 5 = __* *__ multiplied by __ = 15*

Concrete	Pictorial	Abstract
Real objects / manipulatives representing amounts for equal groups (÷): Children divide by sharing into equal groups.  Share 12 counters between 3 boxes.  *12 shared between 6 groups is 2 in each group*	 Draw their own pictures to solve division problems to reinforce language. *12 shared between 3 groups is 4 in each group.*	The division symbol is introduced to the children. *12 ÷ 3 = 4*

Concrete	Pictorial	Abstract
Real objects / manipulatives representing arrays: Understand that multiplication is commutative through making arrays with concrete resources.    Children explore arrays to see that 5 X 2 = 2 X 5 etc.	 Children should be given opportunities to draw their own arrays to represent a multiplication statement. This can be used to identify associated facts.	Arrays should be used to identify all associated facts: *If I know 3 x 4 = 12, then I know:* *4 x 3 = 12* *12 = 4 x 3* *12 ÷ 4 = 3* *3 = 12 ÷ 4* *12 ÷ 3 = 4* *4 = 12 ÷ 3* Also: *5 X 2 = 2 X 5*

Concrete	Pictorial	Abstract
Part part whole:  Emphasis on parts being equal, use Numicon, counters or Cuisenaire with the model. Children to verbalise *'2 x 3 = 6 and 3 x 2 = 6'*	 Draw counters or around Numicon / Cuisenaire to support problem solving.	 *3 x 2 = 6* *2 x 3 = 6*

Concrete	Pictorial	Abstract
Bar models: Emphasis on parts being equal  Use Cuisenaire to represent the multiple parts and whole of the bar model. Cuisenaire can be used on 1cm² paper or alongside a ruler to support understanding of value.	 *2 x 4 = 8* Children draw using squared paper with one square = 1.  *12 ÷ 3 = 4* Children can also use a bar model to 'share' into equal groups.	 *5 x 3 = 15*  *2 x 4 = 8* Children record using bar models.

Concrete	Pictorial	Abstract
Number lines:  Children use Cuisenaire along a ruler to work out product of multiples or to divide a number into equal groups.	 Children can use a number line to find out how many equal groups of 2 can be made from 12. Number line can also be used to consolidate commutative understanding, e.g.  2 x 5 = 5 x 2	**12 ÷ 2 = 6** Children need to recognise the link between division, multiplication and repeated addition. **2 X 6 = 12** **2 + 2 + 2 + 2 + 2 + 2 = 12**

Y3 Multiplication and Division

(NC) Pupils should be taught to:

- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

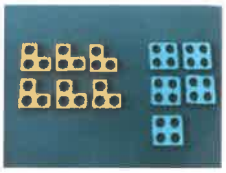
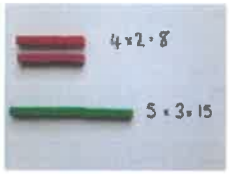
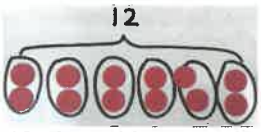
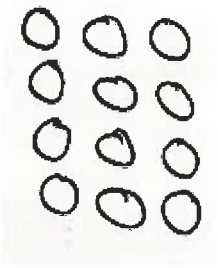
Mental calculation:

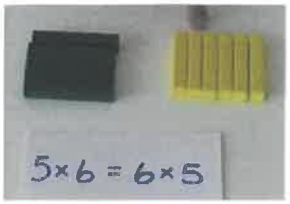
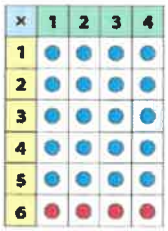
(NC) Pupils should be taught to:

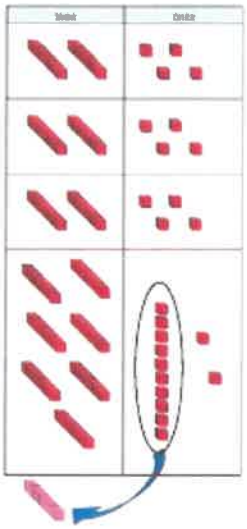
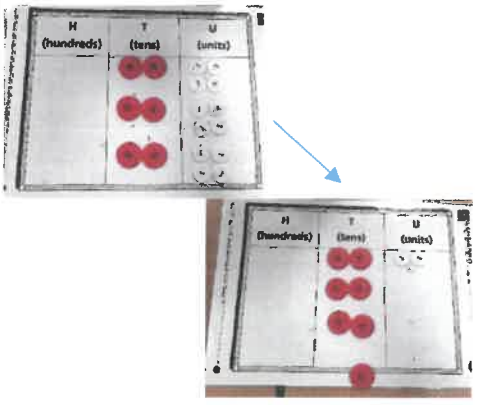
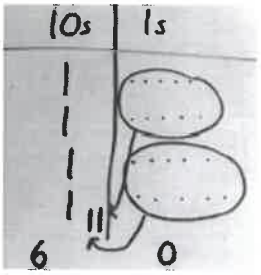
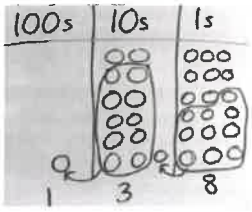
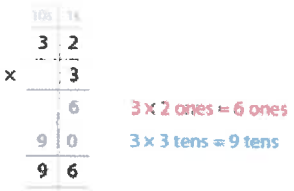
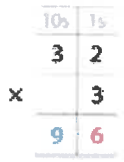
- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables

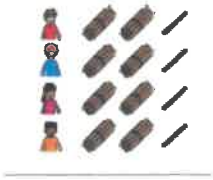
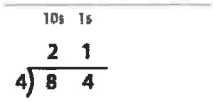
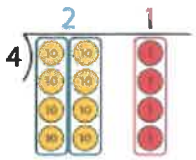
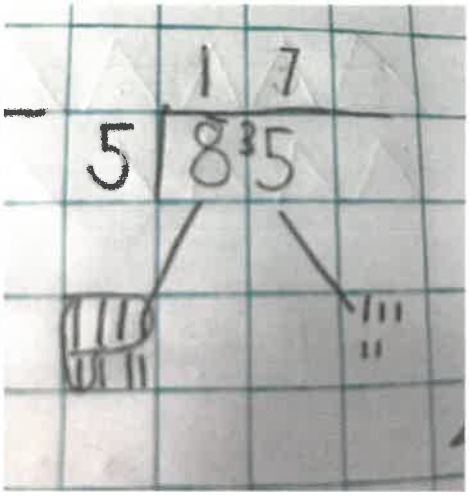
Number facts:

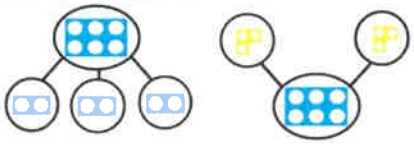
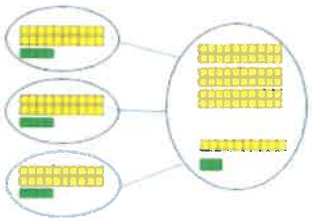
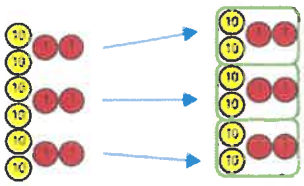
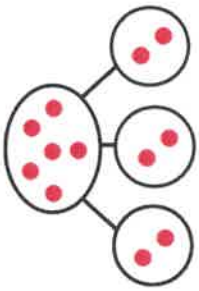
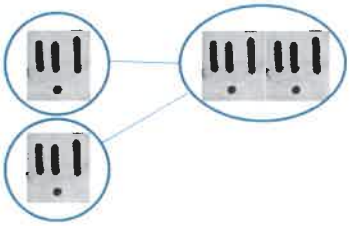
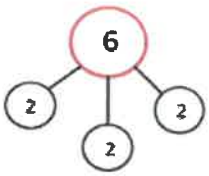
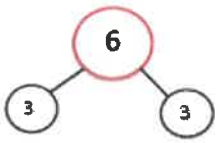
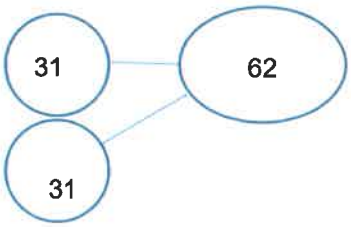
Know x and associated ÷ facts for 3 times table
Know x and associated ÷ facts for 4 times table
Know x and associated ÷ facts for 8 times table

Concrete	Pictorial	Abstract
Real objects / manipulatives representing amounts for equal groups (supporting tables and associated facts): Children can make multiple groups of objects to support conceptual understanding of 'tables'.    Children can make equal groups using a variety of concrete resources to support conceptual understanding of 'times tables'  Share 12 counters between 3 boxes.  *12 shared between 6 groups is 2 in each group*  Use Cuisenaire alongside rulers or squared paper, to support counting in multiples tables and associated division facts.	Children draw their own objects to represent multiplication calculations e.g. *'4 x 3 = 12'*  Children draw their own objects in equal groups to represent division calculations e.g. *'15 ÷ 5 = 3'*  Draw their own pictures to solve division and multiplication problems to reinforce language: *12 shared between 3 equal groups is 4 in each group.* *5 lots of 3 is 15.*	*__ X __ = __* *__ lots of 5 = __* *__ multiplied by __ = 15* *If I know 3 x 4 = 12, then I know:* *4 x 3 = 12 12 = 4 x 3* *12 ÷ 4 = 3 3 = 12 ÷ 4* *12 ÷ 3 = 4 4 = 12 ÷ 3* The division symbol is introduced to the children. *12 ÷ 3 = 4*

Concrete	Pictorial	Abstract
Real objects / manipulatives representing arrays (supporting tables and associated facts): Understand that multiplication is commutative through making arrays with concrete resources.    Children explore arrays to see that 4 X 2 = 2 X 4 etc.	 Children should be given opportunities to draw their own arrays to represent a multiplication statement. This can be used to identify associated facts. Pupils learn the *Distributive Law of multiplication* to secure known facts: *Finding adjacent multiples – array chart:*  6×4=5×4+4	Arrays should be used to identify all associated facts: *If I know 3 x 4 = 12, then I know:* *4 x 3 = 12* *12 = 4 x 3* *12 ÷ 4 = 3* *3 = 12 ÷ 4* *12 ÷ 3 = 4* *4 = 12 ÷ 3* Also: *5 X 2 = 2 X 5* *3 x 4 = (2 x 4) + 4*

Concrete	Pictorial	Abstract
Written method for x: **Diennes** and **PV counters** to model column method:  **24 x 3 = 62**  Use diennes initially, then move to PV counters (once conceptually understanding of PV is secure). Use these to represent 'exchanging' when regrouping for x.	 **15 x 4 = 60**  **23 x 6 = 138** Draw diennes initially, then move to PV counters (once conceptually understanding of PV is secure). Use these to represent 'exchanging' when regrouping for x as shown.	Use expanded column initially with those who need this step before moving on to standard short multiplication, where there is no regrouping: 3 × 32 = ?   • 3 x 2 ones = 6 ones *'Write "6" in the ones column.'* • 3 x 3 tens = 9 tens *'Write "9" in the tens column.'* 2 3 X 6 1 3 8 1 Move to using standard written method alongside concrete and pictorial representation, identifying where regrouping and exchanging has occurred, how this is represented in the abstract form and how it relates to the concrete / pictorial.

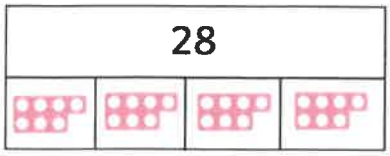
Concrete	Pictorial	Abstract
Written method for ÷: **Diennes** and **PV counters** to model short division (bus shelter) method:   *'8 tens' divided by 4 = 2 tens* *'4 ones' divided by 4 = 1 one* This example uses bunches of straws to represent 'tens' and single straws to represent 'ones'. These could be exchanged for diennes and the process done in the same way.  Move to PV counters (once conceptually understanding of PV is secure).	 	Language to describe division within division problems: ***'...divided into groups of...'*** *e.g.* *'Fifteen divided into groups of five is equal to three.'* ***'...divided between...'*** *e.g.* *Twenty divided between five is equal to four each.'* ***'...divided by...'*** *e.g.* *'Thirty divided by ten is equal to three.'* Calculation represented without drawings (more able may be able to move to this, once place value and regrouping is secure): 98 ÷ 7 becomes 1 4 7 9 2 8 Answer: 14

Concrete	Pictorial	Abstract
Part part whole:  Emphasis on parts being equal, use Numicon, counters or Cuisenaire with the model. Supporting tables facts. Children to verbalise *'2 x 3 = 6 and 3 x 2 = 6'* Extend to diennes then PV counters when looking at 2 digit numbers x 1 digit number.  *24 x 3 = 72*  *66 ÷ 3 = 22*	 Draw counters to support learning tables facts, e.g. *3 x 2 = 6.*  Draw diennes to support 2 digit numbers x 1 digit number, e.g. 62 ÷ 2 = 31.	 *3 x 2 = 6*  *2 x 3 = 6*  *If I know 31 x 2 = 62, then I know:* *31 x 2 = 62* *62 = 31 x 2* *2 x 31 = 62* *62 = 2 x 31* *62 ÷ 2 = 31* *31 = 62 ÷ 2* *62 ÷ 31 = 2* *2 = 62 ÷ 31*

Concrete

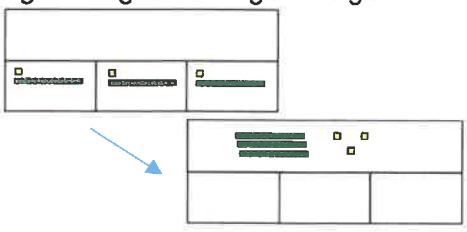
Bar Model:

Emphasis on parts being equal, use Numicon, counters or Cuisenaire with the model to support tables facts:

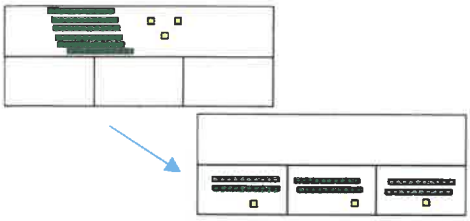


7 x 4 = 28
28 ÷ 4 = 7

Moving on to using bar models for 2 digit x 1 digit and 2 digit ÷ 1 digit:



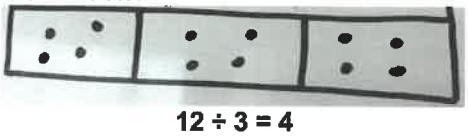
11 x 3 = 33



63 ÷ 3 = 21

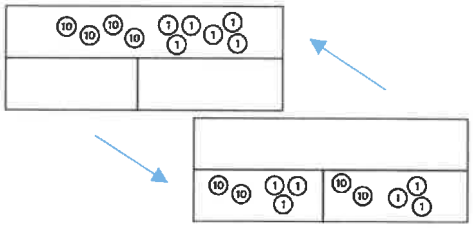
Pictorial

Use bar models to support learning of tables facts:



12 ÷ 3 = 4
4 x 3 = 12

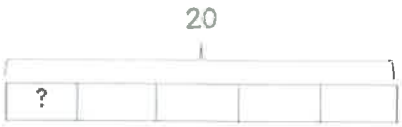
Moving on to using a bar models for 2 digit x 1 digit and 2 digit ÷ 1 digit, using **diennes** or **PV** counters to represent numbers:



46 ÷ 2 = 23
2 x 23 = 46

Abstract

Using the bar to represent problems for x and ÷, using the inverse should support problem solving, so commutative understanding should be secure:

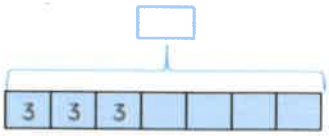


20 ÷ 5 = ?

?

23 x 6 = ?

This can also be used to challenge more able with multistep problems:



Concrete

Number line:

Use Cuisenaire alongside a ruler to introduce number lines for x and ÷, used for tables facts initially:

8 x 3 = 24

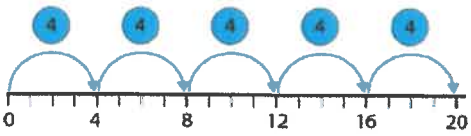
X on a numbers line:
Start at 0, then add cuisenaire to represent the multiples, identifying the total by reading the ruler.

24 ÷ 8 = 3

Use in reverse for ÷:
Start at the dividend on the ruler, work towards 0 with Cuisenaire to represent the multiples of the divisor. Answer (quotient) = number or Cuisenaire used. This can also be extended to identifying 'remainders', once children have mastered using this to solve division with no remainders.

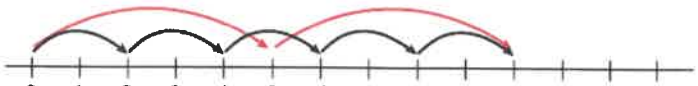
Pictorial

Using the methods explainged in the 'concrete', use a nemberline to support calculating x and ÷. Pupils should draw jumps forwards for x or backwards, towards 0, for ÷:

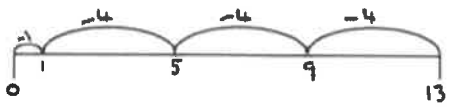


5 x 4 = 20

Number line can also be used to consolidate commutative understanding:



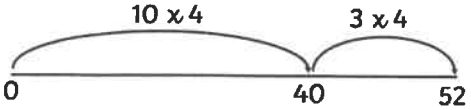
2 x 5 = 5 x 2



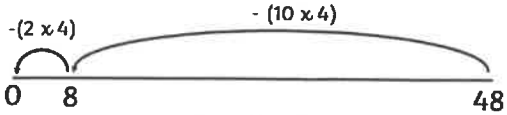
13 ÷ 4 = 3 r1

NB Remainders can be looked at once pupils are confident in using this method for division without remainders.

Moving on to using a number line for 2 digit x 1 digit and 2 digit ÷ 1 digit:



13 x 4 = 52



48 ÷ 4 = 12

Y4 Multiplication and Division

(NC) Pupils should be taught to:

- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

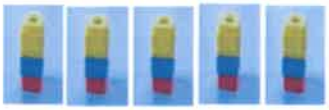
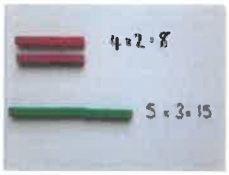
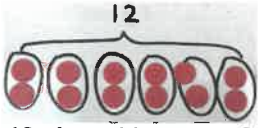
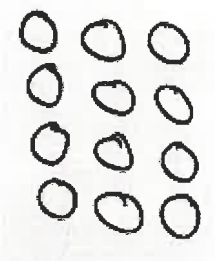
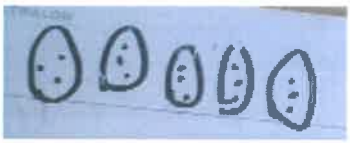
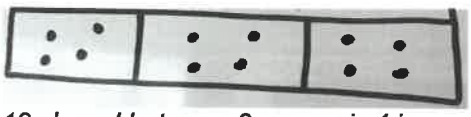
Mental calculation:

(NC) Pupils should be taught to:

- recall multiplication and division facts for multiplication tables up to 12 × 12
- recognise and use factor pairs and commutativity in mental calculations

Number facts:

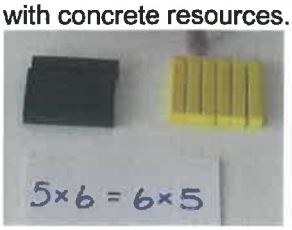
Know x and associated ÷ facts for 9 times table
Know x and associated ÷ facts for 11 times table
Know x and associated ÷ facts for 6 times table
Know x and associated ÷ facts for 12 times table
Know x and associated ÷ facts for 7 times table
Know x and associated ÷ facts for all times table (up to 12x12)

Concrete	Pictorial	Abstract
Real objects / manipulatives representing amounts for equal groups (supporting tables and associated facts):     Children can make equal groups using a variety of concrete resources to support conceptual understanding of 'times tables'  Share 12 counters between 3 boxes.  *12 shared between 6 groups is 2 in each group*	 Children draw their own objects to represent calculations e.g. *'4 x 3 = 12'*  Children draw their own objects to represent calculations e.g. *'15 ÷ 5 = 3'* Draw their own pictures to solve division problems to reinforce language.  *12 shared between 3 groups is 4 in each group.*	*__ X __ = __* *__ lots of 5 = __* *__ multiplied by __ = 15* *If I know 3 x 4 = 12, then I know:* *4 x 3 = 12* *12 = 4 x 3* *12 ÷ 4 = 3* *3 = 12 ÷ 4* *12 ÷ 3 = 4* *4 = 12 ÷ 3* The division symbol is introduced to the children. *12 ÷ 3 = 4*

Concrete

Real objects / manipulatives representing arrays (supporting tables and associated facts):

Understand that multiplication is commutative through making arrays with concrete resources.



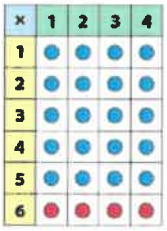
Children explore arrays to see that 4 X 2 = 2 X 4 etc.

Pictorial

Children should be given opportunities to draw their own arrays to represent a multiplication statement. This can be used to identify associated facts.

Pupils learn the *Distributive Law of multiplication* to secure known facts:

Finding adjacent multiples – array chart:



6 × 4 = 5 × 4 + 4

Abstract

Arrays should be used to identify all associated facts:

If I know 3 x 4 = 12, then I know:

4 x 3 = 12 *12 = 4 x 3*
12 ÷ 4 = 3 *3 = 12 ÷ 4*
12 ÷ 3 = 4 *4 = 12 ÷ 3*

Also:

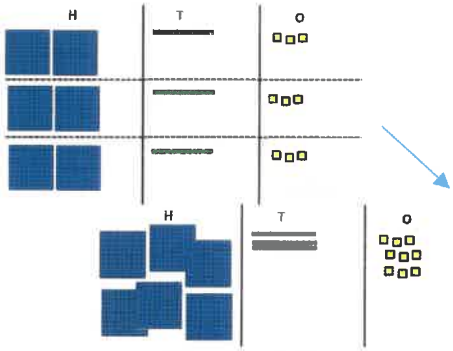
5 X 2 = 2 X 5

3 x 4 = (2 x 4) + 4

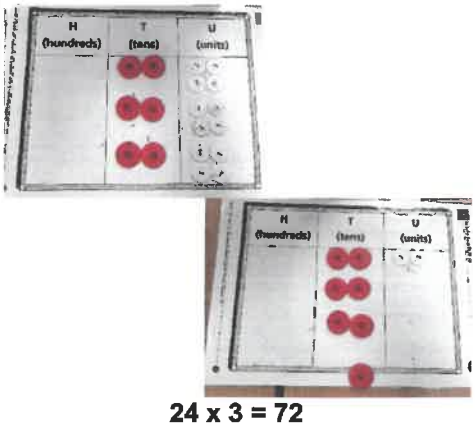
Concrete

Written method for x:

Diennes and **PV counters** to model column method:



213 x 3 = 639

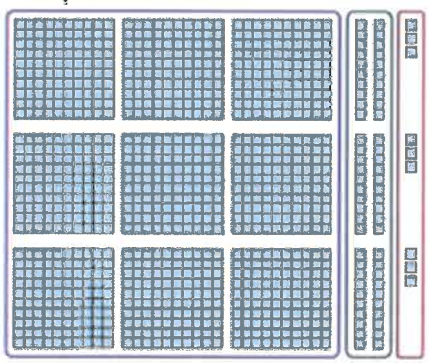


24 x 3 = 72

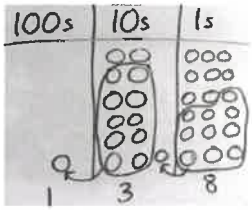
Use diennes initially, then move to PV counters (once conceptually understanding of PV is secure). Use these to represent 'exchanging' when regrouping for x.

Pictorial

Diennes representation:



323 x 3 =969



23 x 6 = 138

Draw diennes initially, then move to PV counters (once conceptually understanding of PV is secure). Use these to represent 'exchanging' when regrouping for x as shown above, but with 3 digit numbers as well..

Abstract

Use expanded column initially with those who need this step before moving on to standard short multiplication, where there is no regrouping:

Multiplication algorithm – expanded layout:

	100s	10s	1s	
	3	2	3	
×			3	
			9	3 × 3 ones = 9 ones
		6	0	3 × 2 tens = 6 tens
	9	0	0	3 × 3 hundreds = 9 hundreds
	9	6	9	

Multiplication algorithm – compact layout:

	3	2	3
×			3
	9	6	9

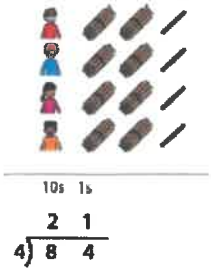
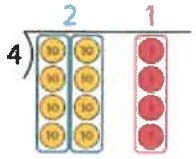
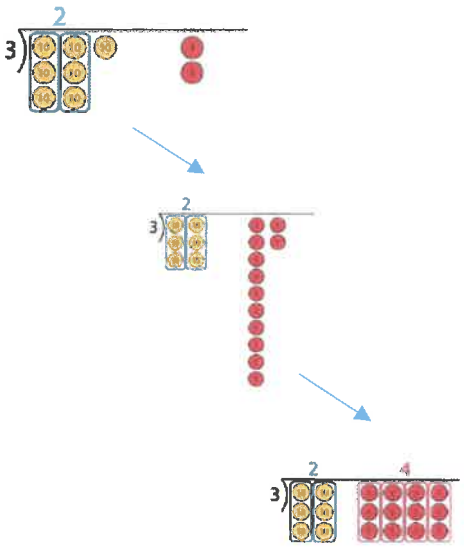
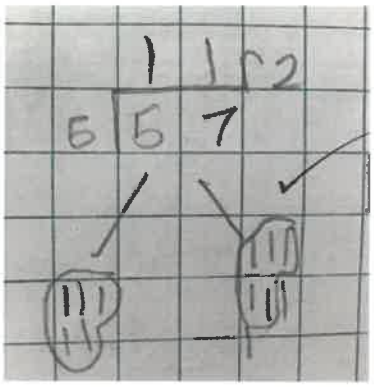
- *'His friend's bookcase holds nine hundred and sixty-nine books.'*

Move to using standard written method alongside concrete and pictorial representation, identifying where regrouping and exchanging has occurred, how this is represented in the abstract form and how it relates to the concrete / pictorial.

342 × 7 becomes

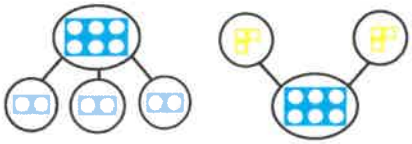
	3	4	2
×			7
2	3	9	4
	2	1	

Remove concrete/pictorial when/where appropriate.

Concrete	Pictorial	Abstract
Written method for ÷: **Diennes** and **PV counters** to model short division (bus shelter) method:  *'8 tens' divided by 4 = 2 tens* *'4 ones' divided by 4 = 1 one* This example uses bunches of straws to represent 'tens' and single straws to represent 'ones'. These could be swapped for **diennes** and the process done in the same way. Moving to using PV counters: 84÷4 =21  Then with remainders, showing regrouping and exchanges: 72÷3=24  	 	Vocabulary to be used alongside sentence stems to discuss bus shelter / short division. quotient divisor dividend Using short division without concrete or pictorial, once confident with conceptual understanding. Initially with 2 digit dividend, then moving to 3 digit dividends. 98 ÷ 7 becomes 1 4 7 9 8 2 Answer: 14 Remainders are left as shown below at this stage: 432 ÷ 5 becomes 8 6 r 2 5 4 3 2 3 Answer: 86 remainder 2

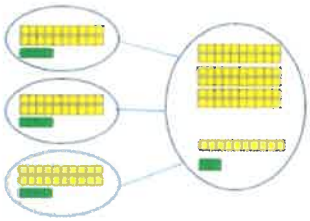
Concrete

Part part whole:

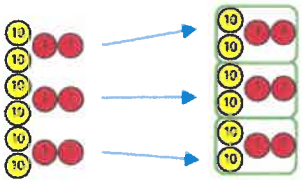


Emphasis on parts being equal, use Numicon, counters or Cuisenaire with the model. Supporting tables facts. Children to verbalise *'2 x 3 = 6 and 3 x 2 = 6'*

Extend to diennes then PV counters when looking at 2 digit numbers x 1 digit number.

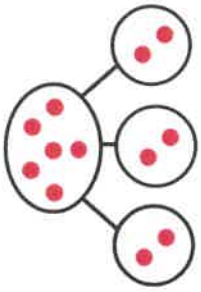


24 x 3 = 72

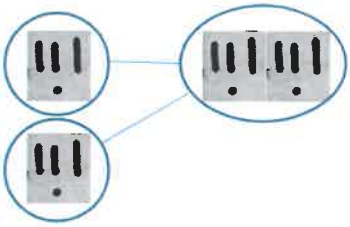


66 ÷ 3 = 22

Pictorial

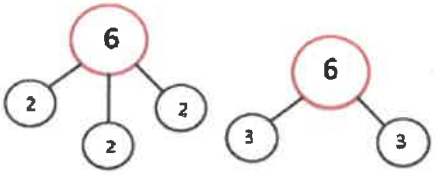


Draw counters to support learning tables facts, e.g. *3 x 2 = 6.*

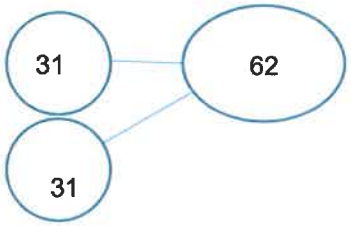


Draw diennes to support 2 digit numbers x 1 digit number, e.g. 62 ÷ 2 = 31.

Abstract



3 x 2 = 6 *2 x 3 = 6*



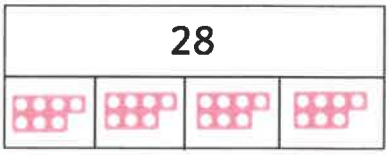
If I know 31 x 2 = 62, then I know:

31 x 2 = 62	*62 = 31 x 2*
2 x 31 = 62	*62 = 2 x 31*
62 ÷ 2 = 31	*31 = 62 ÷ 2*
62 ÷ 31 = 2	*2 = 62 ÷ 31*

Concrete

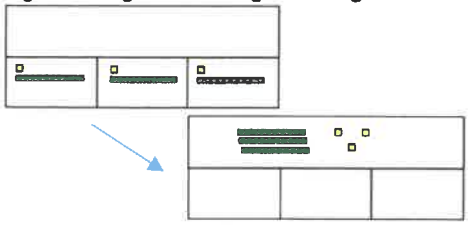
Bar Model:

Emphasis on parts being equal, use Numicon, counters or Cuisenaire with the model to support tables facts:

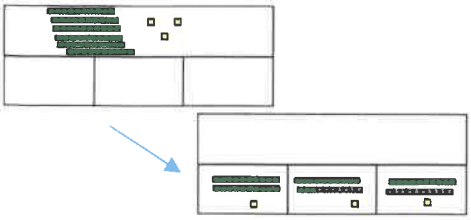


7 x 4 = 28
28 ÷ 4 = 7

Moving on to using bar models for 2 digit x 1 digit and 2 digit ÷ 1 digit:



11 x 3 = 33



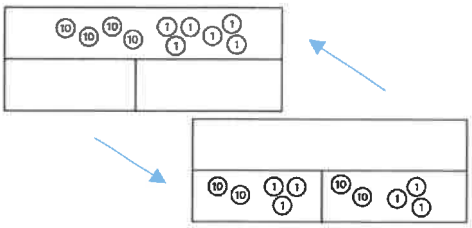
63 ÷ 3 = 21

Pictorial

Use bar models to support learning of tables facts:

12 ÷ 3 = 4
4 x 3 = 12

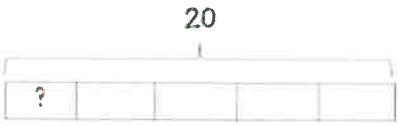
Moving on to using a bar models for 2 digit x 1 digit and 2 digit ÷ 1 digit, using **diennes** or **PV** counters to represent numbers:



46 ÷ 2 = 23
2 x 23 = 46

Abstract

Using the bar to represent problems for x and ÷, using the inverse should support problem solving, so commutative understanding should be secure:

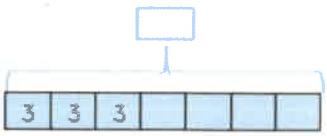


20 ÷ 5 = ?

?

23 x 6 = ?

This can also be used to challenge more able with multistep problems:



Concrete	Pictorial	Abstract

Number line:

Use Cuisenaire alongside a ruler to introduce number lines for x and ÷, used for tables facts initially:

8 x 3 = 24

X on a numbers line:

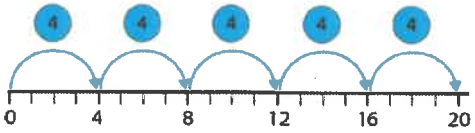
Start at 0, then add cuisenaire to represent the multiples, identifying the total by reading the ruler.

24 ÷ 8 = 3

Use in reverse for ÷:

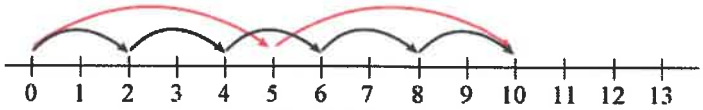
Start at the dividend on the ruler, work towards 0 with Cuisenaire to represent the multiples of the divisor. Answer (quotient) = number or Cuisenaire used. This can also be extended to identifying 'remainders', once children have mastered using this to solve division with no remainders.

Using the methods explainged in the 'concrete', use a nemberline to support calculating x and ÷. Pupils should draw jumps forwards for x or backwards, towards 0, for ÷:

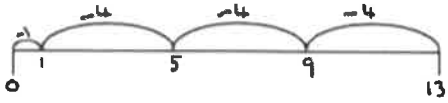


5 x 4 = 20

Number line can also be used to consolidate commutative understanding:



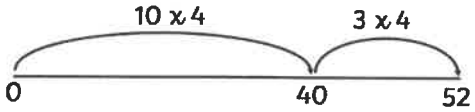
2 x 5 = 5 x 2



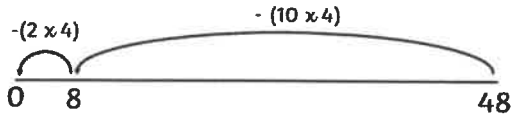
13 ÷ 4 = 3 r1

NB Remainders can be looked at once pupils are confident in using this method for division without remainders.

Moving on to using a number line for 2 digit x 1 digit and 2 digit ÷ 1 digit:



13 x 4 = 52



48 ÷ 4 = 12

Y5 and 6 Multiplication and Division

(NC) Pupils should be taught to (Y5 Y6):

- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- recognise and use square numbers and cube numbers, and the notation for squared (2) and cubed (3)
- solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.
- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- identify common factors, common multiples and prime numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations

Mental calculation:

(NC) Pupils should be taught to (Y5 Y6):

- multiply and divide numbers mentally drawing upon known facts
- perform mental calculations, including with mixed operations and large numbers

Number facts:

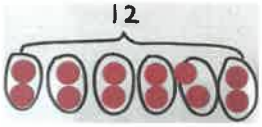
Know x and associated ÷ facts for all times tables, including vocab and symbol 'squared' (e.g. 8^2)
Powers of 10, 100 (e.g. 30 x 8 and 240 ÷ 80)
Repeated x (e.g. 3 x 4 x 5)
Decimals (e.g. 0.3 x 6 and 1.2 ÷ 4)

Concrete

Real objects / manipulatives representing amounts for equal groups (supporting known facts):

Children can make equal groups using a variety of concrete resources to support conceptual understanding. This moves on from previous learning as counters and dienes represent tenths and hundredths, e.g.
Use bar models and part part whole with PV counters as well as those shown here.

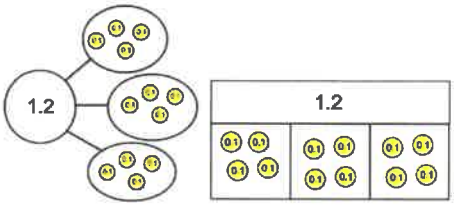
0.4 x 3 = 1.2
Each counter represents 1/10 (0.1).



1.2 ÷ 6 = 0.2
Each counter represents 1/10 (0.1).

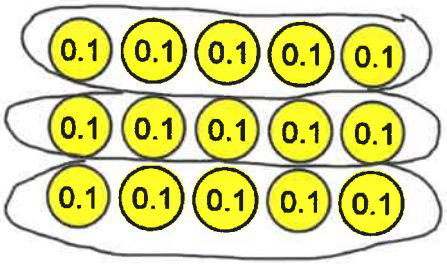
Pictorial

Use bar models and part part whole with drawn PV counters.



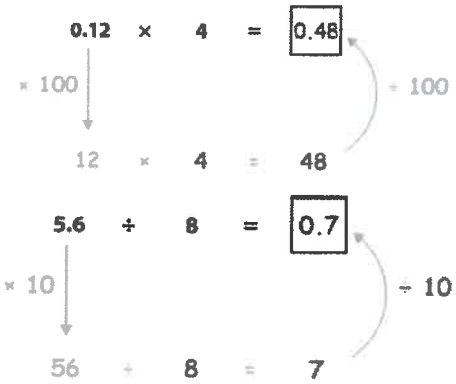
0.4 x 3 = 1.2 or 1.2 ÷ 3 = 0.4
Each counter represent 1/10 (0.1)

Children also draw their own objects to represent calculations e.g. *'0.5 x 3 = 1.5'*



Abstract

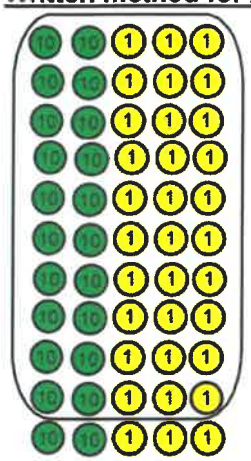
Using known facts and understanding of place value to solve x and ÷ calculations involving decimals.



In these examples the grey recording represents the mental calculation(s) and thinking.

Concrete	Pictorial	Abstract

Written method for x:

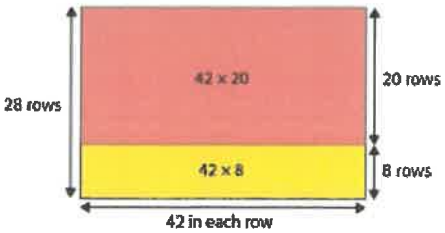


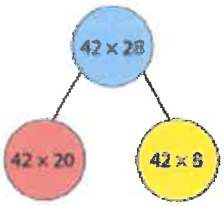
23 x 11 = 253

Partition into rows of tens and ones:

'10 rows of 23 = 230'
'1 row of 23 = 23'

Look at this alongside abstract written method.






```
  42
 X28
 336
  4
 840
1176
 4
```

Remove concrete/pictorial when/where appropriate.

Concrete	Pictorial	Abstract

Written method for ÷:

Use PV counters (both concrete and pictorial) in a similar way to that shown in Y4, for short division, to represent the calculation with those who need additional support.

Interpret the remainder as appropriate for the problem/question, for both short division involving 4 digit dividends (Y5) and long division problems (Y6).

```
      2  8  r 12
   ┌──────────
15 | 4  3  2
     3  0  0
   ──────────
     1  3  2
     1  2  0
   ──────────
        1  2
```

48 ÷ 15 = 28 remainder 12

Moving on to converting the remainder to a fraction:

```
      2  8
   ┌──────────
15 | 4  3  2
     3  0  0     15×20
   ──────────
     1  3  2
     1  2  0     15×8
   ──────────
        1  2
```

$\frac{12}{15} = \frac{4}{5}$

48 ÷ 15 = 28 $\frac{4}{5}$

Then, moving to converting the remainder to a decimal:

```
      2  8 · 8
   ┌──────────
15 | 4  3  2 · 0
     3  0  ↓
   ──────────
     1  3  2
     1  2  0
   ──────────
        1  2  0
        1  2  0
   ──────────
              0
```